# Mastering Calculus 1

### **1. Preliminaries**

#### 1.1. Fundamental Concepts

* **1.1.1.** **Numbers and Operations**
  + Natural Numbers, Integers, Rational and Irrational Numbers
  + Arithmetic Operations and Properties
* **1.1.2.** **Algebraic Expressions**
  + Simplifying Expressions
  + Factoring Techniques
* **1.1.3.** **Equations and Inequalities**
  + Solving Linear and Quadratic Equations
  + Systems of Equations
  + Inequalities and Absolute Values

#### 1.2. Functions and Their Graphs

* **1.2.1.** **Definition and Notation of Functions**
  + Domain and Range
  + Vertical Line Test
* **1.2.2.** **Types of Functions**
  + Polynomial Functions
  + Rational Functions
  + Exponential and Logarithmic Functions
  + Trigonometric Functions
* **1.2.3.** **Transformations of Functions**
  + Shifts, Reflections, Stretching, and Compressing
* **1.2.4.** **Inverse Functions**
  + Definition and Computation
  + Graphical Interpretation

#### 1.3. Mathematical Modeling

* **1.3.1.** **Real-World Applications**
  + Setting Up Mathematical Models
* **1.3.2.** **Interpreting Graphs and Data**

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### **2. Limits and Continuity**

#### 2.1. The Concept of a Limit

* **2.1.1.** **Intuitive Understanding of Limits**
* **2.1.2.** **Numerical and Graphical Approaches**
* **2.1.3.** **One-Sided Limits**

#### 2.2. Calculating Limits

* **2.2.1.** **Limit Laws**
* **2.2.2.** **Algebraic Techniques**
  + Factoring
  + Rationalization
* **2.2.3.** **Special Trigonometric Limits**

#### 2.3. Infinite Limits and Limits at Infinity

* **2.3.1.** **Vertical Asymptotes**
* **2.3.2.** **Horizontal Asymptotes**
* **2.3.3.** **End Behavior of Functions**

#### 2.4. Continuity

* **2.4.1.** **Definition of Continuity at a Point**
* **2.4.2.** **Types of Discontinuities**
* **2.4.3.** **Continuity on Intervals**
* **2.4.4.** **Intermediate Value Theorem**

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### **3. Derivatives**

#### 3.1. Introducing the Derivative

* **3.1.1.** **Tangent Lines and Rates of Change**
* **3.1.2.** **Definition of the Derivative**
  + Limit of the Difference Quotient

#### 3.2. Techniques of Differentiation

* **3.2.1.** **Derivative as a Function**
* **3.2.2.** **Basic Differentiation Rules**
  + Constant Rule
  + Power Rule
  + Constant Multiple Rule
  + Sum and Difference Rules
* **3.2.3.** **Product and Quotient Rules**
* **3.2.4.** **Chain Rule**
  + Differentiating Composite Functions
* **3.2.5.** **Implicit Differentiation**
* **3.2.6.** **Higher-Order Derivatives**

#### 3.3. Derivatives of Specific Functions

* **3.3.1.** **Polynomial and Rational Functions**
* **3.3.2.** **Trigonometric Functions**
* **3.3.3.** **Exponential and Logarithmic Functions**
* **3.3.4.** **Inverse Trigonometric Functions**
* **3.3.5.** **Hyperbolic Functions**

#### 3.4. Applications of the Derivative

* **3.4.1.** **Related Rates**
* **3.4.2.** **Linear Approximations and Differentials**
* **3.4.3.** **Motion Along a Line**
  + Position, Velocity, and Acceleration

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### **4. Applications of Derivatives**

#### 4.1. Analyzing Functions Using Derivatives

* **4.1.1.** **Critical Points and Extrema**
  + Local (Relative) Extrema
  + Absolute (Global) Extrema
* **4.1.2.** **Mean Value Theorem**
* **4.1.3.** **Increasing and Decreasing Functions**
  + First Derivative Test

#### 4.2. Concavity and Curve Sketching

* **4.2.1.** **Concavity and Inflection Points**
* **4.2.2.** **Second Derivative Test**
* **4.2.3.** **Graphing Functions Using Derivatives**

#### 4.3. Optimization Problems

* **4.3.1.** **Setting Up Optimization Problems**
* **4.3.2.** **Solving Optimization Problems**

#### 4.4. Newton's Method

* **4.4.1.** **Approximating Roots**
* **4.4.2.** **Convergence Criteria**

#### 4.5. L'Hospital's Rule

* **4.5.1.** **Indeterminate Forms**
* **4.5.2.** **Applying L'Hospital's Rule**

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### **5. Integration**

#### 5.1. Antiderivatives and Indefinite Integrals

* **5.1.1.** **Basic Antiderivative Rules**
* **5.1.2.** **Initial Value Problems**

#### 5.2. Area and the Definite Integral

* **5.2.1.** **Approximating Area Under a Curve**
  + Riemann Sums (Left, Right, Midpoint)
* **5.2.2.** **Definite Integral as the Limit of a Sum**
* **5.2.3.** **Properties of the Definite Integral**

#### 5.3. Fundamental Theorem of Calculus

* **5.3.1.** **Part I: Connection Between Differentiation and Integration**
* **5.3.2.** **Part II: Evaluating Definite Integrals**

#### 5.4. Techniques of Integration

* **5.4.1.** **Substitution Rule**
* **5.4.2.** **Integrating Trigonometric Functions**

#### 5.5. Applications of Definite Integrals

* **5.5.1.** **Net Change Theorem**
* **5.5.2.** **Areas Between Curves**
* **5.5.3.** **Average Value of a Function**

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### **6. Advanced Topics in Calculus I**

#### 6.1. Further Applications of Integration

* **6.1.1.** **Volume by Slicing (Introduction)**
  + Disk and Washer Methods
* **6.1.2.** **Arc Length (Introduction)**

#### 6.2. Differential Equations (Basics)

* **6.2.1.** **Slope Fields**
* **6.2.2.** **Separable Differential Equations**
* **6.2.3.** **Exponential Growth and Decay Models**

#### 6.3. Improper Integrals (Introduction)

* **6.3.1.** **Integrals over Infinite Intervals**
* **6.3.2.** **Integrals of Unbounded Functions**

#### 6.4. Parametric Equations and Polar Coordinates (Introduction)

* **6.4.1.** **Parametric Equations**
  + Calculus with Parametric Curves
* **6.4.2.** **Polar Coordinates**
  + Graphing in Polar Coordinates

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### **7. Appendices**

#### A.1. Mathematical Notation and Symbols

* Common Symbols Used in Calculus

#### A.2. Review of Trigonometric Identities

* Fundamental Identities
* Double-Angle and Half-Angle Formulas

#### A.3. Proofs of Key Theorems

* Limit Laws
* Derivation of Differentiation Rules

#### A.4. Answers to Selected Exercises

* Solutions to Practice Problems

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### **Additional Resources**

* **Glossary of Terms**
* **Index**

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This comprehensive table of contents covers all fundamental topics in Calculus I, starting from basic concepts and progressing to more advanced topics. Each section is designed to build upon the previous ones, ensuring a solid understanding of calculus principles from beginner to advanced levels. Whether you're just starting out or looking to deepen your knowledge, this guide provides a structured pathway to mastering Calculus I.

#math/calculus